Mapping the space of skills: An approach for comparing embodied sensorimotor organizations

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***Abstract*— This article presents a mathematical framework based on information theory to compare temporally-extended embodied sensorimotor organizations. Central to this approach is the notion of *configuration*: a set of distances between information sources, statistically evaluated for a given time span. Because information distances capture *simultaneously* effects of physical closeness, intermodality, functional relationship and external couplings, a configuration characterizes an embodied interaction with a particular environment. In this approach, collections of skills can be mapped in a unified space as configurations of configurations. This article describes these different abstractions in a formal manner and presents results of preliminary experiments showing how this framework can be used to capture the behavioral organization of an autonomous robot.**

***Index Terms*— Information metric, comparison of skills, relaxation algorithm, self-organizing maps**

I. Introduction

This article is concerned with processes for comparing and making analogies between embodied sensorimotor organizations. This issue is central in the challenging quest for autonomous development [31] as the possibility to find structural similarity between sensorimotor schemas is thought to be crucial for the emergence of higher-level forms of cognition. In particular they permit to consider possibilities of transfer for know-how developed in sensorimotor contexts to more abstract spaces [17], [23]. Important literature exists on how to compare explicit symbolic structure (e.g. [9]), but many authors have argued that generalization and transfer of skills could also be (maybe even more) efficient in the absence of symbolic representation [22].

Given the variability of possible structures that can potentially underly the formation of sensorimotor schemas ([1] (p.36–40), [4], [18], [24], [29]), the approach taken in this article has been made as general as possible. Therefore, it could be applied to a large variety of systems, natural or artificial, for discovering structural similarities between temporally-extended sequences. The starting point of this approach is to suppose that behavioral complexity can be captured by studying the active organization in time of information coming from several *sources*. Sensors mounted on a robot or recording of neural activity are examples of information sources for artificial and living systems, respectively.

Using information sources as a basic modelling unit has several advantages. Models can be framed using the mathematically well-defined tools of information theory. Data coming from various sources (symbol, numerical) can be blended in a unified framework. And finally, the same approach can be used to study both living and artificial systems.

Information theory has historically been mainly concerned with information transmission between a sender and a receiver through a channel [26], [2]. However more recently several lines of research have focused on defining theoretical measures for addressing information integration [30], [27] and information distance between information sources [15], [3]. Crutchfield in particular has shown that the space of information sources can be equipped with a *metric* [3]. Therefore, it is possible to consider a form of *spatial* relationship between sources and to adapt some of the vocabulary and tools of *geometry* to the domain of information theory.

The fact that two information sources are related in terms of information (i.e. that they are close in the information space) means, in a informal way, that knowing the state of one permits to know things about the state of the other. This happens when there is a mutual causal relationship between the two sources or when information coming from both sources result from common causing factors. In our context, this can mean several things. The sources can be physically related and activated by the same localized stimuli. Or they can be functionally related and activated as a result of a particular control pattern. Or they can be only related in time, but still informationally related as the organism interacts with a slowly changing continuous environment. Or they can be related as a result of an external coupling, like in the case where the organism engages in reciprocal interaction with peers. Information is a common currency that permits to blend these multiple factors.

A set of distances between information sources, statistically evaluated for a given time span, specifies a *configura-*

tion. Because information distances capture *simultaneously* effects of physical closeness, intermodality, functional relationship and external couplings, a configuration characterizes an embodied interaction with a particular environment. This leads to interpretation of the emergence of behavioral complexity, as a collection of interrelated configurations in the space of information sources. Configurations can themselves be self-organized in a topological way in a unified space where related configurations are grouped together and independent ones are further apart. In this framework, collections of skills can be viewed as *configurations of configurations*.

The geometrical approach presented in this article is directly inspired by several methods concerning unsupervised map building recently described in the field of artificial intelligence and autonomous robotics. Pierce and Kuipers present a method for building maps of a sensory apparatus out of raw uninterpreted sensory data [21]. This so-called sensory reconstruction method is based on various distances between sensors such as a normalized Hamming distance metric and a frequency metric. Sensors are clustered into subgroups based on their relative distance. The dimensionality of each subgroup can then be computed, related sensors can be projected to form a sensor map. Building on this sensory reconstruction method, Olsson, Nehaniv and Polani [20], [19] have suggested to use the information metric defined by Crutchfield [3] as a more interesting measure of the distance between two information sources. They have conducted experiments with various sensor sets including visual and proprioceptive sensors on an AIBO robot. Related approaches were also investigated by Kuniyoshi's research team [16]. Most of these approaches interpret such sensory reconstruction methods as a way of building maps of sensors in an unsupervised manner. Some of these works make the comparison with somatosensory maps discovered in the brain.

The geometrical approach described in this article extends and, more importantly, reinterprets this method. The sensory reconstruction method is well-adapted to address processes underlying the emergence of behavioral complexity, but it may be misleading to interpret it as a formation of a body map. A particular configuration captures not only aspects of an agent's embodiment, but also reflects the agent's current activities and the situated nature of its interaction with the environment. In particular, a specific configurations appears in the case of couplings with other agents or in cases of remarkable coordination patterns. Integrated views of the schemas organization correspond to a meta-level, namely to *configurations of configurations*. The rest of the paper describes the approach in a more formal manner and presents results of preliminary experiments showing how this framework can be used to capture the behavioral organization of an autonomous robot.

II. Description of the approach

A. Information sources

Let Ω be a system equipped with a set of n information sources $\{X_i\}$. Information sources can be proprioceptives (corresponding to internal states of Ω), heteroceptives (corresponding to information about Ω's environment), or both. Measurements are obtained out of each information source. These measurements typically correspond to elements belonging to an arbitrary number of bins. At each time t, an element x_i corresponds to the information source X_i. The following notation will be used: $X_i(t) = x_i$. As usual with such kind of framework, the choice of the number of bins is an important issue [25]. At any time t, the state of Ω is captured by the vector $X(t)$.

$$X(t) = (X_1(t), X_2(t), \ldots, X_n(t)) \tag{1}$$

Values of $X(t)$ can potentially depend on the environmental context in which the system Ω is placed, the current activity of Ω, as well as its physical and structural organization.

B. Information distance between two information sources

The conditional entropy for two information sources X_i and X_j can be calculated as

$$H(X_j|X_i) = -\sum_{x_i}\sum_{x_j} p(x_i, x_j)\log_2 p(x_j|x_i) \tag{2}$$

where $p(x_j|x_i) = p(x_j, x_i)/p(x_i)$.

$H(X_j|X_i)$ is traditionally interpreted as the uncertainty associated with X_j if the value of X_i is known.

Crutchfield defines the normalized information distance between two information sources as:

$$d(X_j, X_i) = \frac{H(X_i|X_j) + H(X_j|X_i)}{H(X_i, X_j)} \tag{3}$$

d is a metric for the space of information sources [3][1]. This means that it has the three properties of symmetry, equivalence and triangle inequality.

- $d(X, Y) = d(Y, X)$ follows directly from the symmetry of the definition
- $d(X, Y) = 0$ if and only if X and Y are recoding-equivalent (in the sense defined by Crutchfield [3]).
- $d(X, Z) \leq d(X, Y) + d(Y, Z)$

As $H(X_i, X_j) = H(X_i) + H(X_j|X_i)$, $d \leq 1$.
$d = 1$ means that the two sources are independent.

The existence of this metric indicates that the space of information has a topological structure. This permits interesting development such as the continuity of functions on information sources or the convergence of sequences of information sources. However, these properties are not central for the issues discussed in this article.

[1]This is its main advantage compared to mutual information $MI(X_i, X_j) = H(X_i) + H(X_j) - H(X_i; X_j)$

C. Configuration

Let us define a *configuration* as the information distance matrix $\mathbf{D}$ corresponding to the different distances between the information sources X_i

$$\mathbf{D} = \left\{ \begin{array}{c} d(X_1, X_1) \text{ ... } d(X_1, X_n) \\ d(X_2, X_1) \text{ ... } d(X_2, X_n) \\ \dots\dots\dots\dots\dots\dots \\ d(X_n, X_1) \text{ ... } d(X_n, X_n) \end{array} \right\} \tag{4}$$

As $d(X_i, X_i) = 0$, elements of the diagonal are all zero. As $d(X_i, X_j) = d(X_j, X_i)$, $\mathbf{D}$ is symmetrical.

$\mathbf{D}$ summarizes some important aspects about the organization of the information sources of the system Ω, by specifying which sources are related in terms of information and which ones are independent for the context in which the information is gathered. The *geometry* of the mesh corresponding to the different sources is specified by the interdependencies captured in $\mathbf{D}$ and various representations can be created to picture this structure (see below).

D. Configuration of configurations

A configuration can be considered as a point in a configuration space. Various distances between configurations can be envisioned. A simple one is the following:

$$d(\mathbf{M}, \mathbf{N}) = \sqrt{\sum_{kl} (m_{kl} - n_{kl})^2} \tag{5}$$

where m_{kl} and n_{kl} are the components of the kth line and the lth column of respectively the $\mathbf{M}$ and $\mathbf{N}$ matrix.

For a given set of configurations, it is therefore possible to consider the distance matrix of configurations

$$\mathbf{\Delta} = \left\{ \begin{array}{c} d(\mathbf{D_1}, \mathbf{D_1}) \text{ ... } d(\mathbf{D_1}, \mathbf{D_n}) \\ d(\mathbf{D_2}, \mathbf{D_1}) \text{ ... } d(\mathbf{D_2}, \mathbf{D_n}) \\ \dots\dots\dots\dots\dots\dots \\ d(\mathbf{D_n}, \mathbf{D_1}) \text{ ... } d(\mathbf{D_n}, \mathbf{D_n}) \end{array} \right\} \tag{6}$$

As $d(\mathbf{D_i}, \mathbf{D_i}) = 0$, elements of the diagonal are all zero. As $d(\mathbf{D_i}, \mathbf{D_j}) = d(\mathbf{D_j}, \mathbf{D_i}) = 0$, $\mathbf{\Delta}$ is symmetrical.

The configuration of configurations $\mathbf{\Delta}$ captures the organization of sequences of configurations. Related configurations correspond to related activities performed in similar contexts.

E. From distance matrices to maps

1) Representing configurations: Going from relative positions as they are captured by a distance matrix $\mathbf{D}$ to a map representation where points $\{\mathbf{p_i}\}$ can be placed is a constraint-satisfaction problem [21]. Each couple of points $\mathbf{p_i}$ and $\mathbf{p_j}$ should satisfy:

$$||\mathbf{p_i} - \mathbf{p_j}|| = d_{i,j} \tag{7}$$

where $||\mathbf{p_i} - \mathbf{p_j}||$ is the Euclidean distance between the position of the ith and jth point and $d_{i,j}$ the corresponding distance in the matrix $\mathbf{D}$. There are $\frac{n(n-1)}{2}$ equations to satisfy. A set of n points of dimension $n-1$ permits to solve these equation given this set of constraints optimally, but in order to get a lower dimension representation approximation must be taken. Pierce and Kuipers describe a method used by statisticians to determine a good dimensionality for projecting a given set of data [21]. In the rest of the article, two-dimensional projections are used for illustrative purposes although they may not be the optimal ones.

The information contained in $\mathbf{D}$ or $\mathbf{\Delta}$ can be represented in two dimensions using a relaxation algorithm. The algorithm is an iterative procedure of positioning points in a two-dimensional space in such a way that the metric distance between two points in this map is as close as possible to the distance in the distance matrix (other algorithm exist but they use additional information like the relative orientation of connections between points [11], [5]).

The algorithm used consists of an iteration of two simple steps. First, each information source X_i (or configuration $\mathbf{D_i}$) is randomly assigned to a point $\mathbf{p_i}$ on a two-dimensional plane.

1) The force f_i on each point $\mathbf{p_i}$ is computed as:

$$f_i = \sum f_{ij}$$

where

$$f_{ij} = (||\mathbf{p_i} - \mathbf{p_j}|| - d(X_i, X_j)) \frac{(\mathbf{p_j} - \mathbf{p_i})}{||\mathbf{p_j} - \mathbf{p_i}||}$$

2) Each point $\mathbf{p_i}$ is moved according to the force f_i:

$$\mathbf{p_i} = \mathbf{p_i} + \eta f_i$$

where $\eta = 1/n$.

The resulting map partly depends on the initial conditions of the iteration. Several examples of such maps are presented in the following sections.

2) Representing configurations of configurations: For representing configurations of configurations, additional methods based on unsupervised learning can be used, as points (i.e. configurations) are known. Kohonen's self-organizing topological maps are commonly used in artificial neural network models [14]. For instance, they have been used efficiently to model conceptual spaces [8]. Self-organizing maps are suitable for situations where the number of elements to project is open and potentially increasing over time (however, if the number of examples is not restricted, examples should be presented in a repetitive manner). This is why they are used preferentially to map configurations of configurations (see table I for a comparison of relaxation methods and self-organizing maps).

TABLE I
RELAXATION METHOD VS SELF-ORGANIZING TOPOLOGICAL MAPS

	Relaxation method	Self-organizing topological maps
Suitable for systems where only distances are specified (and not points)	Yes	No
Suitable for systems with an open number of prototypes	No	Yes
Can be used to map configurations	Yes	No
Can be used to map configurations of configurations	Yes	Yes

III. EXPERIMENTS WITH AN AUTONOMOUS ROBOT

Experiments of this section involve an autonomous four-legged robot (Sony AIBO ERS-7, dimensions: 180 (W) x 278 (H) x 319 (D) mm). A set of 18 information sources $\{X_1, X_2, ...X_{18}\}$ is used in these experiments. They correspond to distance sensors and proprioceptive position sensors (Table II and figure 1). Each leg has 3 degrees of freedom, as well as the head. Infrared distance sensors are mounted on the head and on the main body[2] (see table II for details of the 18 sensors used in this set of experiments).

TABLE II
SENSORS USED ON THE ROBOT

Number	Name
1-3	distance sensors
4-6	head (proprioceptive sensors)
7-9	right front leg
10-12	right hind leg
13-15	left front leg
16-18	left hind leg

The robot can be programmed to do various kinds of behavior, that range from simple motor skills like walking to integrated forms of behaviors involving more complex sensorimotor coordination like chasing a ball. In its regular autonomous behavior the robot can switch between these various kinds of behavior depending on the evolution of its internal drives and opportunities present in the environment [6], [7]. Although a major issue is to design algorithms permitting to bootstrap new forms of behavior, this section only considers collections of already programmed skills.

In a first experiment, sensory data have been collected from the robot performing a slow walk while moving its head continuously from side to side. During the walk, 1000

[2]The robot has a colour camera mounted above its mouth, electro-static touch sensors, paw sensors, LED lights, all of which are not used in the present experiment but have been exploited in other research conducted with this robot (e.g. [28], [12])

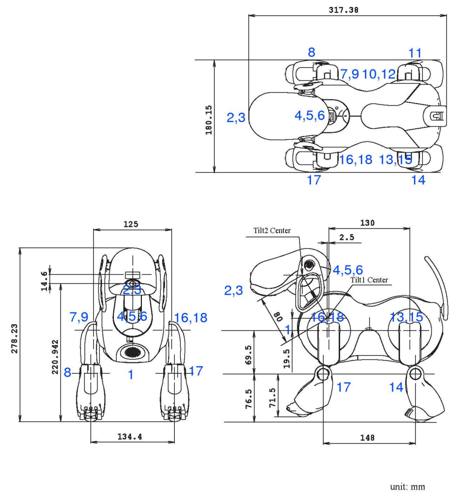

Fig. 1. Top, front, and side view of an AIBO robot with blue numbers indicating the indices for information sources used in experiment (Figure adapted from the Sony AIBO technical manual)

sensor values have been collected for each of these 18 sensors. Figure 2 shows the distance matrix corresponding to this behavior and the associated two-dimensional map. On the map, the arrangement of the information sources corresponds roughly to the sensor distribution on the body of the robot. Distance and head sensors are arranged in the upper right half of the map, the knee joints of all four legs on the lower right of the map and all other leg sensors on the left side. This particular emergent organization results from the physical structure of the robot as well as from the behavioral patterns it conducts in a particular environment. In this particular setting, intrinsic embodiment constraints linking sensor information are probably the most significantly captured (e.g. spatially close similar sensors). However, for other coordination patterns emergent configurations may differ greatly.

In a second experiment, the configurations $\mathbf{D}$ for 11 different types of robot behavior have been calculated. They consisted of five different walking behaviors (walk forward, walk right, ...) and six less oscillating behaviors (cheer, swing, ...). For each behavior, three samples of length 1000 were taken, which corresponds to about seven seconds each. The same sensors as in the previous example were used. The names of the 33 behaviors are listed in table III. The relationship between these 33 configurations has been captured by computing the distance matrix of this configuration of configurations. Corresponding maps have been obtained using a Kohonen self-organizing map with a 10×10 grid (see figure 3) and the relaxation algorithm (see figure 4 b)).

In the distance matrix of figure 4 and in the two maps of figure 3 and 4 one can clearly see two types of structuring. First, configurations of same types of behavior are usually

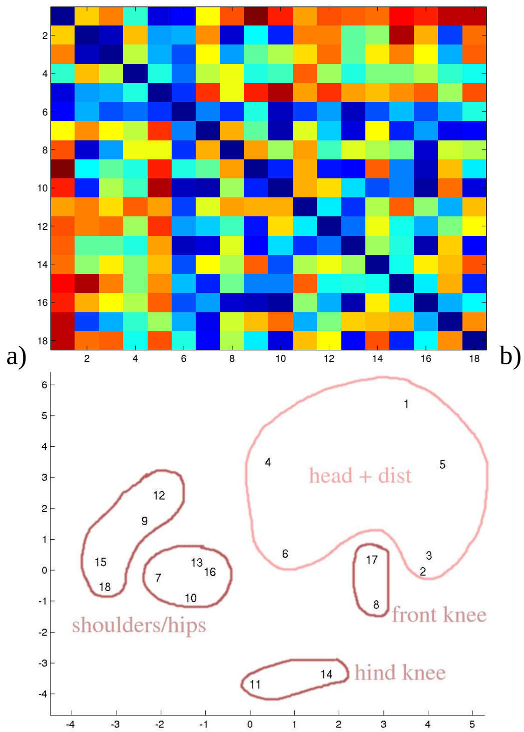

Fig. 2. Walking robot (a) Distance matrix (b) Corresponding two-dimensional map

TABLE III

NAMES OF THE 33 EXAMPLES OF VARIOUS TYPES OF BEHAVIOR

Number	Name
1-3	forward walk
4-6	backward walk
7-9	walk to the right
10-12	turn left
13-15	turn right
16-18	swing
19-21	cheer happy 1
22-24	cheer happy 2
25-27	cheer happy 3
28-30	cheer sad 1
31-33	cheer sad 2

close. Second, at a higher level, configurations for walking behaviors and non-walking behaviors have been differentiated and appear as almost independent subgroups in the maps. Thus, in this example, various situated activities are organized in some forms of hierarchy. Clustering behavior in a hierarchical manner plays an important role for many robotic applications (e.g. [13]). Capturing such kind of emergent organization is also likely to be crucial to permit transfer and analogies between different kinds of behavior.

IV. CONCLUSION

This article presents a mathematical framework for mapping embodied sensorimotor organizations. It is based on two

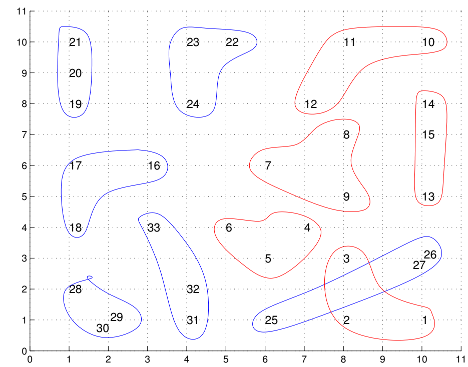

Fig. 3. Map of configurations based on a 10×10 Kohonen SOM. The triples represent same types of behavior. Walking behaviors are marked in red, all other behaviors are marked in blue.

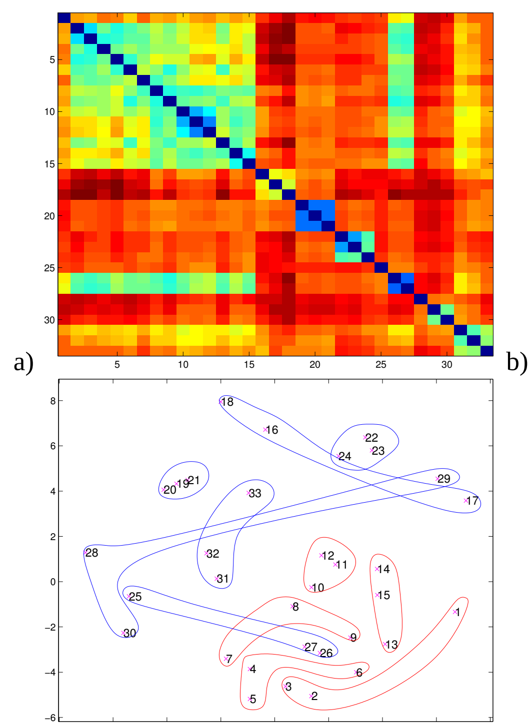

Fig. 4. (a) Distance matrix corresponding to the configuration of the 33 configurations. Small distances are plotted in blue, big distances in red. (b) Map of configurations based on the relaxation algorithm.The triples represent same types of behavior. Walking behaviors are marked in red, all other behaviors are marked in blue.

abstractions characterizing information dependencies in a set of information sources. Configurations reflect context dependent embodied sensorimotor organizations. They capture in a single format information about a physical body structure, particular coordinated actions performed and environmental context. At a second level, configurations of configurations give an integrated view of a collection of skills. This article provides simple illustrations of configurations for a collection of preprogrammed skills.

This approach can easily be extended to account for skills involving couplings between agents. Proprioceptive as well as heteroceptive information can be considered as information sources. In fact, for most organisms, no clear line can be drawn between both. Another set of experiments conducted in the same framework showed how in cases of strong couplings between agents, a "we-centric" space can emerge in which the agent's body structure can be directly mapped onto the structure of an observed body [10].

Although these preliminary results are promising, more work needs to be done to show the relevance of this framework for developmental robotics. Key issues that will be addressed in future work are first, to compare this approach with other ways of making analogies between sensorimotor trajectories and second, to investigate with operant models how emerging structures like configurations play a role in an overall developmental picture.

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